Fall 2016, Math 409, Section 502 **First Midterm (Practice)**

Friday, February 26 2016

Last name:

First name:

UIN:

Signature:

"An Aggie does not lie, cheat or steal or tolerate those who do."

This exam consists of four questions, the total point value of which is 100 points.

The answer to each question must be justified in detail.

The time length of the exam is 50 minutes.

The use of electronic devices, such as cellphones, tables, laptops, and calculators is prohibited.

Good luck!

Problem 1	Problem 2	Problem 3	Problem 4	Total
25	25	25	25	100

Problem 1.

(i) Let A be a non-empty subset of $\mathbb{R}$. When is a number $s \in \mathbb{R}$ called the supremum of A? (Give the definition of the supremum of A) *5 pts.*

(ii) State the completeness axiom of the real numbers. *5 pts.*

(iii) Let A be a non-empty subset of $\mathbb{R}$ and assume that $s = \sup A$ exists. Prove that for every $\varepsilon > 0$ there exists $a \in A$ with $s - \varepsilon < a \leqslant s$. *15 pts.*

Problem 2.

(i) Let $(x_n)_n$ be a real sequence and $a \in \mathbb{R}$. When do we say that $(x_n)_n$ converges to a? (Give the definition of convergence of a real sequence to a real number) *10 pts.*

(ii) Let $x \in \mathbb{R}$. Show that there exists a sequence $(q_n)_n$ in $\mathbb{Q}$ with $\lim_n q_n = x$. *15 pts.*

Problem 3.

(i) Let $(x_n)_n$ be a real sequence. When is $(x_n)_n$ called Cauchy? (Given the definition of a Cauchy real sequence) *7 pts.*

(ii) Show that if $(x_n)_n$ is a real sequence that is Cauchy. then $(x_n)_n$ is convergent. *10 pts.*

(iii) Show that the sequence $(s_n)_n$, with $s_n = \sum_{k=1}^{n} \frac{1}{n}$ for all $n \in \mathbb{N}$, does not converge. *8 pts.*

Problem 4.

(i) **State** the archimedian property of the real numbers. *10 pts.*

(ii) Let a, b be real numbers with $0 < a < b$. Prove that there exists $x_0 \in \mathbb{R}$ so that $(x_0, +\infty) \subseteq \bigcup_{n \in \mathbb{N}} (na, nb)$. *15 pts.*

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