Spring 2017, Math 308, Section 524 **Second Midterm**

Last name: <u>**Sample**</u>

First name:

UIN:

"An Aggie does not lie, cheat or steal or tolerate those who do."

This exam consists of **five** problems, the total point value of which is 100 points.

The answer to each question must be justified in detail.

The time length of this exam is 75 minutes.

The use of electronic devices, such as cellphones, tables, laptops, and calculators is prohibited.

Good luck!

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Total
20	20	20	20	20	100

Problem 1. In each of the following questions choose one appropriate answer. No justification for your answer is necessary. (20 pts in total total)

(i) The differential equation

$$\ln(t)y'' - \frac{11}{t}y' + 4y = \sin(1/t), \quad t > 0$$

is a (4 pts)

(a) linear differential equation (b) non-linear differential equation

and it is (4 pts)

(a) homogeneous. (b) non-homogeneous.

(ii) The characteristic equation of the differential equation

$$y'' + 6y' - 4y = 0$$

is (4 pts)

(a) $2r + r^6 - 1 = 0$. (b) $r^2 + 6r - 4 = 0$. (c) $4r^2 + 6r - 1 = 0$.

(iii) The corresponding homogeneous equation of the differential equation

(NH) $$y'' + 2y' + y = t^2 \sin(7t)$$

is (4 pts)

(a) $y'' + 2y' + y - t^2 \sin(7t) = 0$ (b) $y'' + 2y' + y = 0$

and an appropriate guess for a particular solution of (NH) is (4 pts)

(a) $Y = A\sin(7t) + Bt\sin(7t) + Ct^2\sin(7t)$.
(b) $Y = At^2\sin(7t) + Bt^2\cos(7t)$.
(c) $Y = A\sin(7t) + Bt\sin(7t) + Ct^2\sin(7t) + D\cos(7t) + Et\cos(7t) + Ft^2\cos(7t)$.

Problem 2.

(i) Find the general solution of (8 pts)

(∗) $$y'' - 2y' + 26y = 0.$$

(ii) Find the general solution of $t^2y'' - ty' + 26y = 0$, $t > 0$. (6 pts)

Hint: use the change of variable $x = \ln(t)$

(iii) Solve the i.v.p. (6 pts)

$$t^2y'' - ty' + 26y = 0,\ t > 0,$$
$$y(e^{\pi/5}) = -1,\ y'(e^{\pi/5}) = -6e^{-\pi/5}.$$

Problem 3.

(i) It is given that $y_1 = e^t$ is a solution to the differential equation 10 pts

(H) $$y'' - \left(1 - \frac{\cos(t) + \sin(t)}{\cos(t) - \sin(t)}\right) y' - \left(\frac{\cos(t) + \sin(t)}{\cos(t) - \sin(t)}\right) y = 0,$$
$$0 < t < \pi/4.$$

Find a fundamental set of solutions for (H).

(ii) Find the general solution of 10 pts

(NH) $$y'' - \left(1 - \frac{\cos(t) + \sin(t)}{\cos(t) - \sin(t)}\right) y' - \left(\frac{\cos(t) + \sin(t)}{\cos(t) - \sin(t)}\right) y = \cos(t) - \sin(t),$$
$$0 < t < \pi/4.$$

Problem 4. Given that $y_1 = e^t$, $y_2 = e^{2t}$ form a fundamental set of solutions for the corresponding homogeneous problem,
(i) find a particular solution of 9 pts

$$y'' - 3y' + 2y = \frac{130}{7}\sin(3t), \tag{NH1}$$

(ii) find a particular solution of 7 pts

$$y'' - 3y' + 2y = e^{2t}, \tag{NH2}$$

(iii) find the general solution of 4 pts

$$y'' - 3y' + 2y = \frac{130}{7}\sin(3t) + e^{2t}. \tag{NH3}$$

Problem 5. If a is a non-zero number, recall that $\mathscr{L}\left\{e^{at}\right\} = \dfrac{1}{s-a}$ for $s > a$. Also, recall that for a function f, under certain regularity conditions,

$$\mathscr{L}(f') = s\mathscr{L}(f) - f(0) \text{ and } \mathscr{L}(f'') = s^2\mathscr{L}(f) - sf(0) - f'(0).$$

(i) Find the inverse Laplace transform of (12 pts)

$$F(s) = \frac{5s - 14}{s^2 - 5s + 4}.$$

(ii) Use the Laplace transform to solve the initial value problem. (8 pts)

$$y'' - 5y' + 4y = 0, \quad y(0) = 5,\ y'(0) = 11.$$