Spring 2017, Math 308, Section 524 **First Midterm**

Last name: **Sample**

First name:

UIN:

"An Aggie does not lie, cheat or steal or tolerate those who do."

This exam consists of **five** problems, the total point value of which is 100 points.

The answer to each question must be justified in detail.

The time length of this exam is 75 minutes.

The use of electronic devices, such as cellphones, tables, laptops, and calculators is prohibited.

Good luck!

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Total
20	20	20	20	20	100

Problem 1. In each of the following questions choose one appropriate answer. No justification for your answer is necessary. (20 pts in total total)

(i) The differential equation

$$y''' + e^t y^4 = 0.$$

is a (4 pts)

(a) linear differential equation (b) non-linear differential equation

and it is a (4 pts)

(a) first (b) second (c) third (d) fourth

order differential equation.

(ii) The differential equation

$$\frac{dy}{dx} = \frac{x^4 + 3x^2 + 2}{e^y + \sin(y)}$$

is a/an (4 pts)

(a) separable differential equation (b) autonomous differential equation.

(iii) The differential equation

$$\frac{dy}{dx} = -y^4 - 2y^2$$

is a/an (4 pts)

(a) linear differential equation (b) autonomous differential equation

and any solution to this differential equation satisfying $y(0) = 0.5$ is (4 pts)

(a) upwards concave
(b) downwards concave
(c) initially upwards concave and eventually downwards concave.

Problem 2. Consider the initial value problem (20 pts in total)

$$y' + \frac{2t}{t^2+1} y = -\sin(t),$$
$$y(0) = y_0$$

Show that $\lim_{t\to\infty} (y(t) - \cos(t)) = 0$. (20 pts)

Problem 3. An object of mass m is dropped from rest into a bottomless pit containing a medium that offers resistance of $k|v|$, where v is the velocity of the object and k is a positive constant. The force of gravity acting on the object is gm where g is the acceleration due to gravity.

(i) Use Newton's second law and the chain rule to set up a first order separable differential equation that relates the velocity of the object to the distance by which it has dropped. (5 pts)

(ii) Find the distance by which the object has dropped when it has reached velocity $v = \frac{mg}{2k}$. (15 pts)

Problem 4. Consider the differential equation (20 pts in total)

$$(*) \qquad \frac{dy}{dx} = (y-1)(y-2)^2.$$

(i) Find the equilibrium solutions of $(*)$. (5 pts)

(ii) Draw the phase line and find the stability type of the equilibrium solutions. (7 pts)

(iii) Draw the integral curves (i.e. the graphs of several solutions y), taking into consideration the monotonicity and concavity of the solutions. (8 pts)

Problem 5. Consider the differential differential equation (20 pts in total)

$$(*) \qquad y^2 - 2e^{-x}\sin(x)y + \left(2y + 2e^{-x}\cos(x)\right)\frac{dy}{dx} = 0$$

(i) Check whether this is an exact differential equation. (6 pts)

(ii) If it is not exact, find an appropriate integrating factor to make it exact.(7 pts)

(iii) Solve the initial value problem given by $(*)$ and the initial condition $y(0) = 1$. For full points, write an **explicit** solution of y. (7 pts)

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