Fall 2017, Math 304 **First Midterm**

Sample

Last name:

First name:

UIN:

"An Aggie does not lie, cheat or steal or tolerate those who do."

This exam consists of **ten** problems, the total point value of which is 100 points.

The answer to each question must be **justified in detail**, unless otherwise specified.

The time length of this exam is 50 minutes.

The use of electronic devices, such as cellphones, tables, laptops, and calculators is prohibited.

Good luck!

Prb. 1	Prb. 2	Prb. 3	Prb. 4	Prb. 5	Prb. 6	Prb. 7	Prb. 8	Prb. 9	Prb. 10	Total
10	10	10	10	10	10	10	10	10	10	100

Problem 1. Let A be the 3×3-matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

(i) Write down the transpose A^T. *3 pts.*

No justification is necessary for the answers to the following questions.

(ii) Is A symmetric? *1 pt.*

(iii) Is A diagonal? *1 pt.*

(iv) Is A upper triangular? *1 pt.*

(v) Is A lower triangular? *1 pt.*

(vi) What is $\det(A)$? *3 pt.*

Problem 2. Let B be a 4×4-matrix such that $\det(B) = 2$.

(i) Is B singular or non-singular? No justification is necessary. *2 pt.*

(ii) Is the transpose B^T singular or non-singular? *2 pts.*

(iii) Is B^2 singular or non-singular? *2 pts.*

(iv) Solve the equation $Bx = \mathbf{0}$. *4 pts.*

Problem 3.

(i) Let A be a non-singular matrix. Prove that $\det(A^{-1}) = 1/\det(A)$. *4 pt.*

(ii) If A is an $n \times n$ matrix and λ is a scalar prove that $\det(\lambda A) = \lambda^n \det(A)$. *3 pt.*

(iii) If A is an non-singular $n \times n$ matrix so that $A^{-1} = -A$ prove that n must be even. *3 pt.*

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Problem 4.

(i) If A and B are $n \times n$ matrices with $AB = I$ prove that neither A nor B can be singular. *6 pt.*

(ii) If A and B are $n \times n$ non-zero matrices with $AB = \mathbf{0}$, prove that both A and B must be singular. *4 pt.*

Problem 5. Consider the linear system

$$(\triangle) \qquad \begin{array}{ccccccc} x_1 & & -x_3 & +2x_4 & = & -2 \\ -x_1 & +x_2 & +2x_3 & -3x_4 & = & 7 \\ 2x_1 & & +2x_3 & -4x_4 & = & 8 \end{array}$$

(i) Write the augmented coefficient matrix of this system. *3 pt.*
(ii) Write an equivalent coefficient matrix so that the left part is in reduced row echelon form. *4 pt.*
(iii) Identify the lead variable and the free variables of $(\triangle)$. *1 pt.*
(iv) Solve $(\triangle)$ *2 pt..*

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Problem 6. Given a number $0 \leq \theta < 2\pi$ consider the 3×3 matrix

$$R = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix}$$

(i) Verify that R is invertible with $R^{-1} = R^T$. *7 pt.*

(ii) Solve the equation $R\mathbf{x} = \mathbf{e}_2$, where $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. *3 pt.*

Problem 7.

(i) Compute the inverse of the matrix *6 pts.*

$$D = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(ii) Find a 2×2 matrix X that satisfies the equation $AX + B = X$ where *4 pt.*

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 0 & -2 \\ 1 & 4 & 0 \\ -2 & 1 & -3 \end{bmatrix}.$$

Problem 8.

(i) Consider the following subsets of $\mathbb{R}^3$:

$$S_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - c = b \right\}, \quad \text{and} \quad S_2 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a^2 + b^2 + c^2 = 1 \right\}$$

Which of the above are subspaces and which are not? *6 pt.*

(ii) Do the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ form a spanning set of $\mathbb{R}^3$? *4 pt.*

Problem 9.
Consider the matrix
$$A = \begin{bmatrix} 1 & -2 & -2 & 1 & 1 \\ -2 & 4 & 4 & 1 & 3 \\ 1 & -2 & -2 & 2 & 1 \end{bmatrix}.$$
(i) Find the null space of A. *8 pt.*
(ii) Solve the equation $Ax = \mathbf{b}$ if x_0 is a known solution in the cases: *2 pt.*
$$(1)\ \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix},\ x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{and} \quad (2)\ \mathbf{b} = \begin{bmatrix} -1 \\ 5 \\ 0 \end{bmatrix},\ x_0 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

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Problem 10.

(i) Compute the determinant of the matrix *7 pts.*

$$A = \begin{bmatrix} 0 & -1 & -2 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 3 \end{bmatrix}.$$

(ii) Are the vectors

$$v_1 = \begin{bmatrix} 0 \\ -1 \\ -2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, v_4 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

linearly independent? *3 pt.*