Fall 2016, Math 409, Section 502 **Ninth Assignment**

Due Friday, April 15 (at the beginning of class)

Exercise 1. Let $f : (a, b) \to \mathbb{R}$ be a differentiable function, $x_0 \in (a, b)$, and $\lambda \in \mathbb{R}$. If $\lim\limits_{x \to x_0} f'(x) = \lambda$, prove that f' is continuous at x_0 and $f'(x_0) = \lambda$.
Hint: use the intermediate value theorem for derivatives. *3 pts.*

Exercise 2. Prove that for all $x \in [0, +\infty)$ we have

$$0 \leqslant x - \sin(x) \leqslant \frac{x^3}{6}.$$

You may use that if $f(x) = \sin(x)$, then $f'(x) = \cos(x)$, $f''(x) = -\sin(x)$ and $-1 \leqslant \cos(x) \leqslant 1$ for all $x \in \mathbb{R}$.
Hint: use Taylor's formula and $3! = 6$. *4 pts.*

Exercise 3. Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable functions so that $g'(x) \neq 0$ for all $x \in \mathbb{R}$ and and the function $h : \mathbb{R} \to \mathbb{R}$ with $h(x) = f'(x)/g'(x)$ is bounded. If g is uniformly continuous, show that f is uniformly continuous as well.
Hint: use the generalized mean value theorem. *3 pts.*