Fall 2016, Math 409, Section 502 **Eighth Assignment**

Due Friday, April 1 (at the beginning of class)

Exercise 1. A function $f : \mathbb{R} \to \mathbb{R}$ is called periodic, with period $T \in \mathbb{R}$, if for every $x \in \mathbb{R}$ we have $f(x) = f(x+T)$. If f is such a function and f is continuous, prove that it is uniformly continuous. *2 pts.*

Exercise 2. Let $f : [0,\infty) \to \mathbb{R}$ be a continuous function and assume that the limit $\lim_{x\to\infty} f(x)$ exists and it is a real number. Prove that f is uniformly continuous. *3 pts.*

Exercise 3. Let $f : [0,1] \to \mathbb{R}$ be a continuous function and let $\varepsilon > 0$. Prove that there exists $n \in \mathbb{N}$ so that for $k = 1,\ldots,n$ we have *3 pts.*

$$\sup\left\{f(x) : \frac{k-1}{n} \leqslant x \leqslant \frac{k}{n}\right\} - \inf\left\{f(x) : \frac{k-1}{n} \leqslant x \leqslant \frac{k}{n}\right\} < \varepsilon.$$

Exercise 4. Let $n \in \mathbb{Z}$, and $f : (0,+\infty) \to \mathbb{R}$ be the function $f(x) = x^n$. Prove that $f'(x) = nx^{n-1}$ for all $x \in (0,+\infty)$. *2 pts.*