Fall 2016, Math 409, Section 502 **Sixth Assignment**
Due Friday, March 11 (at the beginning of class)

Exercise 1. Use **only the definition of the limit of a function** to solve the following: *2 pts.*

(i) if $a \in \mathbb{R}$, prove that $\lim_{x \to a} x = a$,
(ii) if $a \in \mathbb{R}$, prove that $\lim_{x \to a} |x| = |a|$,
(iii) if $a \in \mathbb{R}$, prove that $\lim_{x \to a} x^2 = a^2$.

Exercise 2. Using only properties of limits proved so far, evaluate the limit *2 pts.*

$$\lim_{x \to 3} \frac{x^2 - 2x - 3}{x^3 - 9x}.$$

Exercise 3. Let $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$ be functions, and a, L, and M be real numbers. Assume that there exists an open interval I, containing a, so that $f(x) \neq L$ for all $x \in I \setminus \{a\}$. Then, if $\lim_{x \to a} f(x) = L$, and $\lim_{x \to L} g(x) = M$, prove that $\lim_{x \to a} g(f(x)) = M$. *2 pts.*

Exercise 4. Let I be an open interval of $\mathbb{R}$, $a \in I$ and f be a functions whose domain contains $I \setminus \{a\}$. Assume that for every sequence $(x_n)_n$ in $I \setminus \{a\}$, that converges to a, the sequence $(f(x_n))_n$ converges to some real number, **that may depend on the sequence**. Prove that there exists $L \in \mathbb{R}$ with $\lim_{x \to a} f(x) = L$. *2 pts.*

Exercise 5. Let $n \in \mathbb{N}$ and let $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial of degree n (i.e. $a_n \neq 0$). Prove that $\lim_{x \to +\infty} P(x) = +\infty$, if $a_n > 0$, or $\lim_{x \to +\infty} P(x) = -\infty$, if $a_n < 0$. *2 pts.*