Fall 2016, Math 409, Section 502 **Third Assignment**

Due Friday, February 12 (at the beginning of class)

Exercise 1. Find $f[A]$ and $f^{-1}[A]$, where $f : \mathbb{R} \to \mathbb{R}$, in the following cases:

(i) $f(x) = 2 - 3x$ and $A = (-1, 2)$, *2 pts.*

(ii) $f(x) = \cos(x)$ and $A = [0, \infty)$.

Exercise 2. Let X, Y be sets and $f : X \to Y$ be a function. *3 pts.*

(i) If $(B_i)_{i\in I}$ is a collection of subsets of Y prove

$$f^{-1}\left[\bigcup_{i\in I} B_i\right] = \bigcup_{i\in I} f^{-1}\left[B_i\right] \text{ and } f^{-1}\left[\bigcap_{i\in I} B_i\right] = \bigcap_{i\in I} f^{-1}\left[B_i\right].$$

(ii) If D, F are subsets of Y prove

$$f^{-1}\left[D \setminus F\right] = f^{-1}\left[D\right] \setminus f^{-1}\left[F\right].$$

(iii) If A is a subset of X and B is a subset of Y, then

$$f\left[f^{-1}\left[B\right]\right] \subseteq B \text{ and } f^{-1}\left[f\left[A\right]\right] \supseteq A.$$

Exercise 3. Let A and B be sets and assume that A is uncountable. If there exists a function $g : B \to A$ that is onto, prove that B is uncountable. *2 pts.*

Exercise 4. Let A be a finite set and $B \subseteq A$. Show that B is finite.

Hint: prove by induction on n the following statement: if $F \subseteq \{1, \dots, n\}$ and $F \neq \varnothing$, then there exists a natural number $k \leqslant n$ and a bijection $f : F \to \{1, \dots, k\}$. *3 pts.*