Fall 2016, Math 409, Section 502 **Tenth Assignment**

Due Friday, April 22 (at the beginning of class)

Exercise 1. Let $f : (a, b) \to \mathbb{R}$ be a differentiable, bounded and 1-1 function. Prove the following: *4 pts.*

(i) There exist real numbers $c < d$ with $f[(a, b)] = (c, d)$.

(ii) For every $r < s \in (a, b)$ there exists $x \in (r, s)$ with $f'(x) \neq 0$.

(iii) For every $v < w \in (c, d)$ there exists $y \in (v, w)$ so that the function $f^{-1} : (c, d) \to \mathbb{R}$ is differentiable at y.

Exercise 2. Let $F = \{a_1, \ldots, a_n\}$ be a finite subset of $[0, 1]$ and let also $f : [0, 1] \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \in F \\ 0 & \text{if } x \in [0, 1] \setminus F. \end{cases}$$

Use the definition of the Riemann integral to show that f is Riemann integrable and $\int_0^1 f(x)\mathrm{d}x = 0$. *3 pts.*

Exercise 3. Let $f : [a, b] \to \mathbb{R}$ be a continuous function with $f(x) \geqslant 0$ for all $x \in [a, b]$. If f is not the zero function, show that $\int_a^b f(x)\mathrm{d}x > 0$. *3 pts.*