Fall 2016, Math 409, Section 502 **First Assignment**

Due Friday, January 29 (at the beginning of class)

Exercise 1. Using only the field axioms and order axioms of $\mathbb{R}$, prove the statements below. *5 pts.*

(i) If a is a non-zero real number, then its multiplicative inverse is unique.

(ii) If a is a real number, then $a \cdot 0 = 0$.

(iii) If a is a real number, then $-a = (-1) \cdot a$.

(iv) If a, b, c, and d are real numbers so that $a < b$ and $c < d$, then $a + c < b + d$.

(v) If a and b are real numbers with $0 < a < b$, then $0 < 1/b < 1/a$ (here, $1/a$ denotes the multiplicative inverse of a).

Exercise 2. Let $x \in \mathbb{R}$. *2 pts.*

(i) Find all the values of x for which $|3x + 2| < 10$.

(ii) Show that $|x| \leqslant 2$ implies $|x^2 - 1| \leqslant 3|x + 1|$.

Exercise 3. Let x, $y \in \mathbb{R}$. Show that if $x < (1 + \varepsilon)y$ for all $\varepsilon > 0$, then $x \leqslant y$. *3 pts.*