Practice problems for the second midterm exam

Math 251, Fall 2015

Sections 504 & 513

Problem 1. Evaluate the double integral $\iint_R e^x \sin(y) \mathrm{d}A$, where R is the rectangle $[0, \ln(2)] \times [0, \pi]$.

Problem 2. Evaluate the double integral $\iint_R \sin(x+2y) \mathrm{d}A$, where R is the rectangle $[-\pi/4, \pi/4] \times [0, \pi/2]$.

Problem 3. Let E denote the solid that lies above the rectangle $R = [2,3] \times [-1,0]$ and below the elliptic paraboloid $3x^2 + 3y^2 = z$.

(i) Set up a double iterated integral express the volume of E.

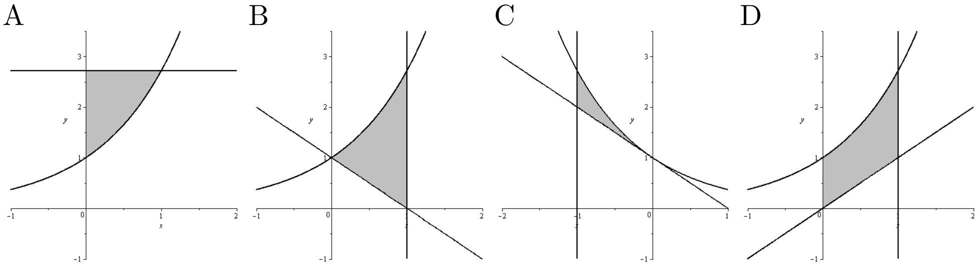
(ii) Find the volume of E.

Problem 4.

(i) Evaluate the iterated integral $\int_0^1 \int_x^{e^x} 3y^2 \mathrm{d}y \mathrm{d}x$.

(ii) Identify the region of integration.

A B C D



(iii) Express the region of integration as a region of type I.

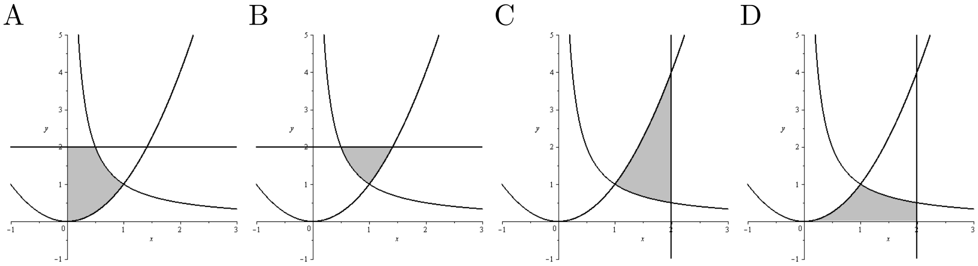
Recall: A region of type I is of the form

$$D = \{(x,y) | a \leqslant x \leqslant b, g_1(x) \leqslant y \leqslant g_2(x)\}.$$

Problem 5. Let D be the region of the plane bounded by $y = 1/x$, $y = x^2$ and $x = 2$.

(i) Identify the region of integration.

A B C D



(ii) Express D as a region of type I.

(iii) Evaluate the double integral $\iint_D 2x^2y\,\mathrm{d}A$.

Problem 6. Let D be the region of the plane bounded by $y = \sqrt{x}$, $y = 1$ and $x = 0$.

(i) Express D as a region of type I.
(ii) Express D as a region of type II.
Recall: A region of type II is of the form

$$D = \{(x, y)|c \leqslant y \leqslant d, h_1(y) \leqslant x \leqslant h_2(y)\}.$$

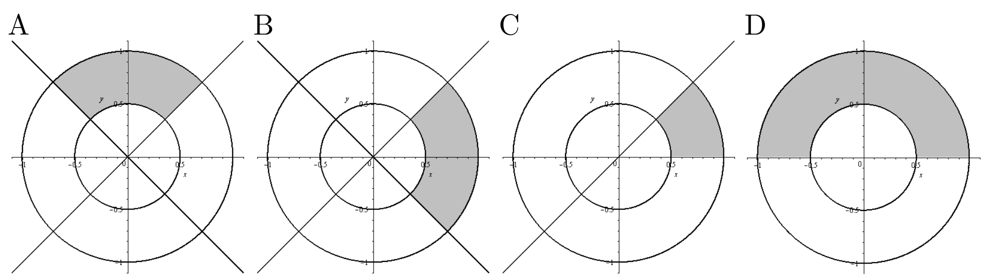
(iii) Evaluate the double integral $\iint_D e^{-y^3}\mathrm{d}A$.
Hint: choose the appropriate expression of D.

Problem 7. Let E denote the solid in the first octant, below the elliptic paraboloid $z = x^2 + 3y^2$ and bounded by the plane $x + y = 1$.

(i) Set up a double iterated integral expressing the volume of E.
(ii) Find the volume of E.

Problem 8. Let R be the region of the plane inside the circle $x^2 + y^2 = 1$, but outside the circle $x^2 + y^2 = 1/4$ and above the lines $y = x$ and $y = -x$.

(i) Identify the region of integration.



(ii) Express R in polar coordinates.
(iii) Write $\displaystyle\iint_R \frac{x + y}{x^2 + y^2}\mathrm{d}A$ in the form of a double iterated integral with polar coordinates.
(iv) Evaluate $\displaystyle\iint_R \frac{x + y}{x^2 + y^2}\mathrm{d}A$.

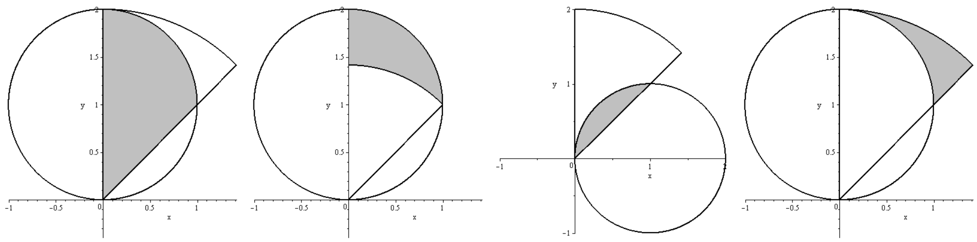
Problem 9. Let E denote the solid above the xy-plane, in the cylinder $x^2+y^2 = 1$ and below the upper sheet of the hyperboloid $-x^2-y^2+z^2 = 1$.

(i) Set up a double iterated integral with polar coordinates expressing the volume of E.
(ii) Find the volume of E.

Problem 10. Let D denote the region inside the polar rectangle defined by $0 \leqslant r \leqslant 2$ and $\pi/4 \leqslant \theta \leqslant \pi/2$, but outside the circle $r = 2\sin(\theta)$.

(i) Identify D.

A B C D



(ii) Express D as polar region of type II.
Recall: a polar region of type II is of the form

$$D = \{(r, \theta) : \alpha \leqslant \theta \leqslant \beta, \rho_1(\theta) \leqslant r \leqslant \rho_2(\theta)\}.$$

(iii) Set up a double iterated integral with polar coordinates expressing the area of D.
(iv) Find the area of D.

Problem 11. Let D denote the disk with center the origin and radius one. A pizza occupies the region D. The inexperienced cook distributed the toppings unevenly and so, the mass density at each point of the pizza is given by $\rho(x, y) = y + 2$.

(i) Set up a double iterated integral in polar coordinates expressing the total mass of the pizza.
(ii) Find the total mass of the pizza.
(iii) Set up a double iterated integrals in polar coordinates expressing the center of mass of the pizza.
(iv) Find the center of mass of the pizza.
Hints: $\sin^2\theta = (1/2)(1 - \cos(2\theta))$, $\int_0^{2\pi} \cos\theta \sin\theta \mathrm{d}\theta = 0$.

Problem 12. Evaluate the triple integral $\iiint_B xyz\mathrm{d}V$, if B is the rectangular box $[0,1] \times [-1,0] \times [1,2]$.

Problem 13.

(i) Evaluate the triple integral $\int_0^1 \int_{x^2}^{x} \int_{x^2+y^2}^{x^2+2y^2} x\mathrm{d}z\mathrm{d}y\mathrm{d}x$.
(ii) Express the region of integration as a region of type 1.

Problem 14. Let E be the solid region above the plane $x - z = 1$, beneath the elliptic paraboloid $z = x^2 + 3y^2$, bounded by the planes $x = 0$, $y = 0$ and $x + y = 1$.

(i) Express E as a solid region of type 1.
(ii) Set up a triple iterated integral which expresses the volume of E.
Recall: the volume of E is $\iiint_E \mathrm{d}V$.
(iii) Evaluate the volume of E.

Problem 15.

(i) Identify the type of surface defined by the equation $1/2 = \sin^2\phi\cos^2\theta$ in spherical coordinates.

(ii) Identify the type of surface defined by the equation $\cos^2(\phi) = \frac{\rho^2-1}{2\rho^2}$ in spherical coordinates.

Problem 16.

(i) Identify the type of surface defined by the equation $z^2 = 1 + r^2$ in cylindrical coordinates.

(ii) Set up a triple iterated integral in cylindrical coordinates that expresses the volume of the solid region E, bounded by $z^2 = 1 + r^2$ and the cylinder $r = 2$.

(iii) Evaluate the volume of E.

Problem 17. Let E be the spherical wedge defined by $\sqrt[3]{\pi/4} \leqslant \rho \leqslant \sqrt[3]{\pi/2}$, $0 \leqslant \theta \leqslant \pi/2$ and $\pi/4 \leqslant \phi \leqslant \pi/2$. A solid is occupying region E and its mass density is described by the function $\varrho(x, y, z) = \sin((x^2 + y^2 + z^2)^{3/2})$.

(i) Set up a triple iterated integral in spherical coordinates that expresses the total mass of the solid.

(ii) Evaluate the total mass of the solid.