

Practice problems for the first midterm exam

Math 251, Fall 2015

Sections 504 & 513

Problem 1.

- (a) Find an equation of the sphere with center $(1, -2, 4)$ and radius 3.
- (b) What is the intersection of the sphere with the yz -plane?

Problem 2. Let $\mathbf{a} = \langle 2, 2, 0 \rangle$ and $\mathbf{b} = \langle 0, 4, 0 \rangle$ be three dimensional vectors.

- (a) Find the angle between the vectors \mathbf{a} and \mathbf{b} .
- (b) Find the area of the triangle defined by the vectors \mathbf{a} and \mathbf{b} .

Problem 3. Find the volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 2, 1, 0 \rangle$, $\mathbf{b} = \langle 1, -1, 2 \rangle$ and $\mathbf{c} = \langle 2, 1, -3 \rangle$.

Problem 4. Find a vector equation **or** parametric equations for the line that passes through the point $(1, -7, 23)$ and is parallel to the line $x = 31 + 9t$, $y = -49 - 29t$, $z = 3 - 93t$.

Problem 5.

- (a) Find a vector equation **or** parametric equations for the line that passes through the point $(0, 13, 2)$ and is perpendicular to the plane $2x - 3y + z = 42$.
- (b) At what point does this line intersect the xy -plane?

Problem 6.

- (a) Find an equation for the plane that contains the point $(-1, 4, 0)$ and is perpendicular to the line $x = 2 + 11t$, $y = -3 - 7t$, $z = -1 + 2t$.
- (b) At what point do the line and plane intersect?

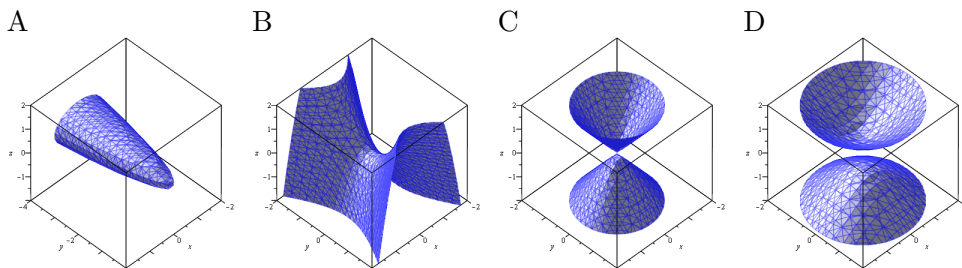
Problem 7. Find an equation for the plane that contains the points $A = (1, 2, 3)$, $B = (2, 3, 4)$ and $C = (6, 5, 4)$.

Problem 8. Find the angle between the planes $x + 2y + z = 19$ and $2x + y - 4z = 42$.

Problem 9. Find the distance from the point $(1, 1, 1)$ to the plane $2x + 3y + 6z = 1$.

Problem 10.

- (a) Identify the type of surface described by the equation $3x^2 + 7z^2 = -y$.
- (b) Choose the figure that corresponds to the surface.

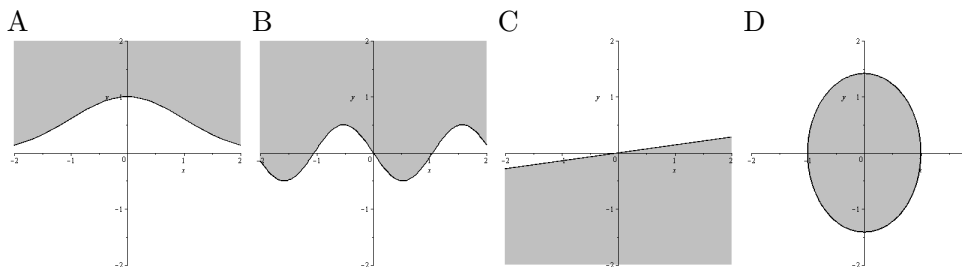


Problem 11. Find a vector equation **or** parametric equations for the tangent line to the space curve of the vector function $\mathbf{r}(t) = \langle \ln t, te^{t-1}, \sqrt{t} \rangle$ at the point $(0, 1, 1)$.

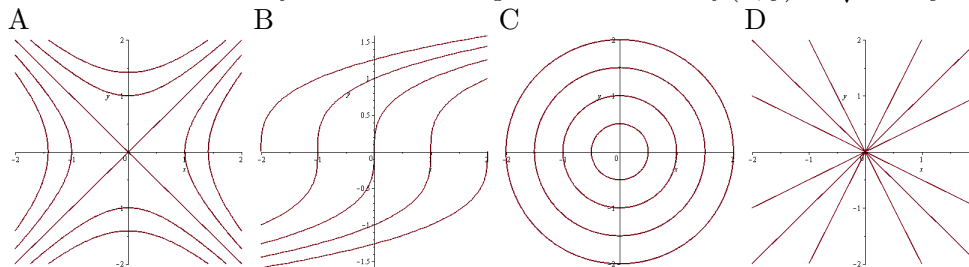
Problem 12. Find the length of the space curve of the vector function $\mathbf{r}(t) = \langle \sin(e^t), \cos(e^t), e^t \rangle$, $0 \leq t \leq \ln(2)$.

Problem 13.

- (a) Find the domain of the function $f(x, y) = \ln(x - 4y)$.
 (b) Choose the figure that corresponds to the domain of f .



Problem 14. Identify the contour map of the function $f(x, y) = \sqrt{x^2 + y^2}$.



Problem 15. Describe the level surfaces of the function $f(x, y, z) = x^2 + 2y^2 + 5z^2$.

Problem 16. If $f(x, y) = e^{xy} + xe^y + 2x^2y^3$, find:

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial x \partial y}.$$

Problem 17. Two sides of a triangle, who form an angle θ , have lengths x and y . If $x = 1\text{m}$ with a possible error of 0.001m , $y = 3\text{m}$ with a possible error of 0.01m and $\theta = \pi/6$ rad with a possible error of 0.1rad , use differentials to estimate the maximum error of the length of the third side of the triangle.
Hint: the length of the third side of the triangle is $\sqrt{x^2 + y^2 - 2xy \cos \theta}$.

Problem 18. If $w = 3x^2 + xy^2 + e^y z$ and $x = sr^2 e^t$, $y = r^2 + st^3$, $z = e^r \sin(st)$, use the chain rule to find $\frac{\partial w}{\partial r}$, $\frac{\partial w}{\partial s}$, $\frac{\partial w}{\partial t}$.

Problem 19. Assume that z is implicitly defined by $\cos(z^2) - x^2 + y^2 = x \sin(z)$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Problem 20. An elliptic pancake is being prepared in a hot pan. The heat of the pan causes the semi-minor axis of the pancake to increase with a rate of 0.01 mm/sec. Unfortunately, the poor quality of the pan and the uneven distribution of the pancake-mix have resulted in the shrinkage of the semi-major axis of the pancake with a rate of 0.02 mm/sec. When the cook realizes this, the semi-minor axis of the pancake is 40mm long whereas the semi-major axis of the pancake is 50mm long. Find the rate of change of the pancake's area, at the moment when the cook makes this realisation.

Hint: if $x =$ (length of semi-minor axis) and $y =$ (length of semi-major axis), then the area of the pancake is πxy .

Problem 21. Find an equation for the tangent plane to the surface

$$z = e^{xy} + 3yx^2 + 1$$

at the point $(0, 0, 2)$.

Problem 22. Let $f(x, y, z) = 4 \sin(xe^y) + \cos(z)$.

- Find ∇f .
- Find the rate of change of f at the point $(\pi/4, 0, \pi)$ in the direction of the vector $\mathbf{u} = \langle 2, 2, 1 \rangle$

Problem 23.

- Find an equation of the plane tangent to the surface

$$e^{1-z} \cos(1-z) = x^2 z^2 + yz^2$$

at the point $(1/2, 3/4, 1)$.

- Find parametric equations of the normal line to the same surface at the same point.

Problem 24. The temperature of the water in a certain region under the surface of the ocean is given by $T(x, y, z) = z^3 + 2x + 2y + \sin(xy) + 32$. A diver is situated at the point $(0, 0, 0)$, where it is inconveniently cold. In which direction should the diver swim to experience the maximum rate of change in temperature? How rapidly is the temperature rising in that direction?

Problem 25. Find the local maxima, local minima and saddle points, if there are any, of the function $f(x, y) = x^4 - 4xy + y^4$.

Problem 26. Find the local maxima, local minima and saddle points, if there are any, of the function $f(x, y) = 2x^2 + 2y^2 + 4/(xy)$.

Problem 27. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 4xy + 2y$ on the set

$$D = \{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}.$$

Problem 28. Find the point on the plane $2x - 2y + 4z - 2 = 0$ that is closest to the point $(3, 1, -4)$.

Problem 29. You need to build a rectangular box made of cardboard. Each of its edges needs to be securely connected to all of its adjacent edges with the use of duct tape. You have a large supply of cardboard, but unfortunately you only have 6 feet of duct tape in your possession. Find the dimensions of the box with the largest possible volume that you can build.

Problem 30 (bonus, non-exam material).

- (a) Find the point on the surface $z - y^2 - 3x^2 = 0$ that has the shortest distance from the plane $-2x + 2y + z + 2 = 0$.
- (b) Find parametric equations for the normal line to the surface at that point.
- (c) Where does this line intersect the plane?
- (d) You find yourself allowed to only move on the surface $z - y^2 - 3x^2 = 0$ whereas your significant other half is only allowed to move on the plane $-2x + 2y + z + 2 = 0$. Where should each of you be situated to be as close to each other as possible?